# Quark intrinsic motion and the link between TMDs and PDFs in covariant approach *

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The relations between TMDs and PDFs are obtained from the symmetry requirement: relativistic covariance combined with rotationally symmetric parton motion in the nucleon rest frame. This requirement is applied in the covariant parton model. Using the usual PDFs as an input, we are obtaining predictions for some polarized and unpolarized TMDs.

The transverse momentum dependent parton distribution functions (TMDs) [1, 2] open the new way to more complete understanding of the quark-gluon structure of the nucleon. We studied this topic in our recent papers [3–5]. We have shown, that requirements of symmetry (relativistic covariance combined with rotationally symmetric parton motion in the nucleon rest frame) applied in the covariant parton model imply the relation between integrated unpolarized distribution function and its unintegrated counterpart. Obtained results are shortly discussed in the first part. Second part is devoted to the discussion of analogous relation valid for polarized distribution functions.

**Unpolarized distribution function**

In the covariant parton model we showed [6], that the parton distribution function $f_1^q(x)$ generated by the 3D distribution $G_q$ of quarks reads:

$$f_1^q(x) = Mx \int G_q(p_0)\delta\left(\frac{p_0+p_1}{M} - x\right)\frac{dp_1 d^2\mathbf{p}_T}{p_0} \tag{1}$$

and that this integral can be inverted

$$G_q\left(\frac{M}{2}x\right) = -\frac{1}{\pi M^3}\left(\frac{f_1^q(x)}{x}\right)'. \tag{2}$$

Further, due to rotational symmetry of the distribution $G_q$ in the nucleon rest frame, the following relations for unintegrated distribution were obtained [5]:

$$f_1^q(x, \mathbf{p}_T) = MG_q\left(\frac{M}{2}\xi\right). \tag{3}$$

After inserting from Eq. (2) we get relation between unintegrated distribution and its integrated counterpart:

$$f_1^q(x, \mathbf{p}_T) = -\frac{1}{\pi M^2}\left(\frac{f_1^q(\xi)}{\xi}\right)'; \qquad \xi = x\left(1 + \left(\frac{p_T}{Mx}\right)^2\right). \tag{4}$$

Now, using some input distributions $f_1^q(x)$ one can calculate transverse momentum distribution functions $f_1^q(x, \mathbf{p}_T)$. As the input we used the standard PDF parameterization [8] (LO at the scale $4GeV^2$). In Fig. 1 we have results obtained from relation (4) for $u$ and $d-$quarks. The right part of this figure is shown again, but in different scale in Fig 2. One can observe the following:

*i)* For fixed $x$ the corresponding $p_T-$ distributions are very close to the Gaussian distributions

$$f_1^q(x, p_T) \propto \exp\left(-\frac{p_T^2}{\langle p_T^2\rangle}\right). \tag{5}$$

*ii)* The width $\langle p_T^2\rangle = \langle p_T^2(x)\rangle$ depends on $x$. This result corresponds to the fact, that in our approach, due to rotational symmetry, the parameters $x$ and $p_T$ are not independent.

*iii)* Figures suggest the typical values of transversal momenta, $\langle p_T^2\rangle \approx 0.01GeV^2$ or $\langle p_T\rangle \approx 0.1GeV$. These values correspond to the estimates based the analysis of the experimental data on structure function $F_2(x, Q^2)$ [5]. They are substantially lower, than the values $\langle p_T^2\rangle \approx 0.25GeV^2$ or $\langle p_T\rangle \approx 0.44GeV$ following e.g. from the analysis of data on

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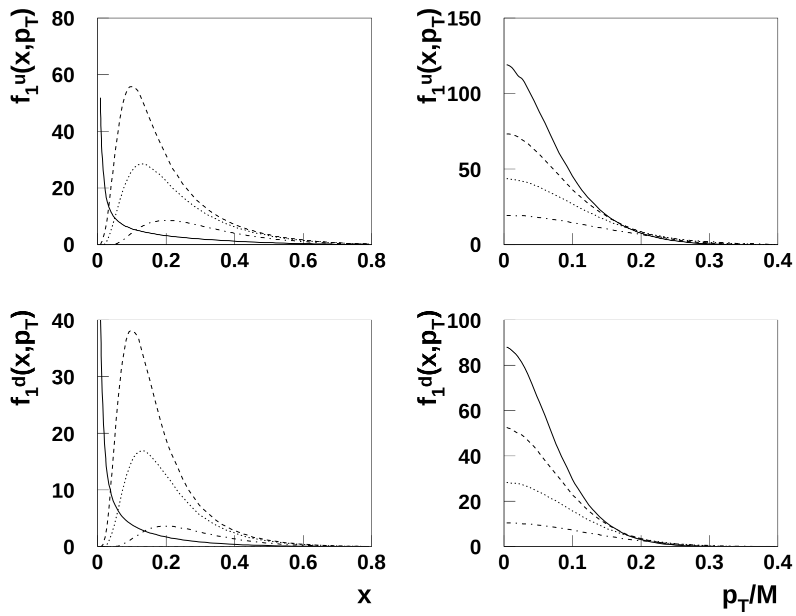


FIG. 1: Transverse momentum dependent unpolarized distribution functions for $u$ (*upper figures*) and $d-$quarks (*lower figures*). **Left part**: dependence on $x$ for $p_T/M = 0.10, 0.13, 0.20$ is indicated by dash, dotted and dash-dot curves; solid curve correspods to the integrated distribution $f_1^q(x)$. **Right part**: dependence on $p_T/M$ for $x = 0.15, 0.18, 0.22, 0.30$ is indicated by solid, dash, dotted and dash-dot curves.

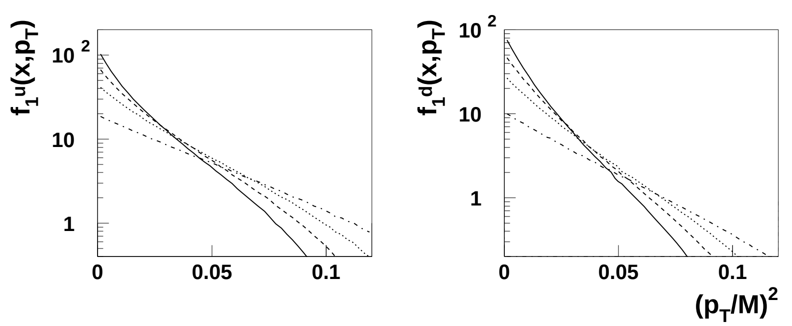


FIG. 2: Transverse momentum dependent unpolarized distribution functions for $u$ and $d-$quarks. Dependence on $(p_T/M)^2$ for $x = 0.15, 0.18, 0.22, 0.30$ is indicated by solid, dash, dotted and dash-dot curves.

the Cahn effect [9] or HERMES data [10]. At the same time the fact, that the shape of obtained $p_T-$ distributions (for fixed $x$) is close to the Gaussian, is remarkable. In fact, the Gaussian shape is supported by phenomenology.

**Polarized distribution functions**

Relation between the distribution $g_1^q(x)$ and its unintegrated counterpart can be obtained in a similar way, however in general the calculation with polarized structure functions is slightly more complicated. First let us remind procedure for obtaining structure functions $g_1, g_2$ from starting distribution functions $G^{\pm}$ defined in [6], Sec. 2, see also the footnote there. In fact the auxiliary functions $G_P, G_S$ are obtained in appendix of the paper [7]. If we assume that $Q^2 \gg 4M^2x^2$, then the approximations

$$|\mathbf{q}| \approx \nu, \qquad \frac{pq}{Pq} \approx \frac{p_0 + p_1}{M} \tag{6}$$

are valid and the equations (A1),(A2) can be with the use of (A3),(A4) rewritten as

$$G_X = \int \Delta G\left(p_0\right) w_X \delta\left(\frac{p_0+p_1}{M}-x\right)\frac{dp_1 d^2\mathbf{p}_T}{p_0}, \qquad X=P,S \tag{7}$$

where

$$\begin{aligned} w_P &= -\frac{m}{2M^2\nu}\frac{-p_1\cos\omega+p_T\cos\varphi\sin\omega}{p_0+m} \\ &\times\left(1+\frac{1}{m}\left(p_0-\frac{-p_1-(-p_1\cos\omega+p_T\cos\varphi\sin\omega)\cos\omega}{\sin^2\omega}\right)\right), \end{aligned} \tag{8}$$

$$\begin{aligned} w_S &= \frac{m}{2M\nu}\left(1+\frac{-p_1\cos\omega+p_T\cos\varphi\sin\omega}{p_0+m}\frac{1}{m}\right. \\ &\times\left.\left(-p_1\cos\omega+p_T\cos\varphi\sin\omega-\frac{-p_1-(-p_1\cos\omega+p_T\cos\varphi\sin\omega)\cos\omega}{\sin^2\omega}\cos\omega\right)\right). \end{aligned} \tag{9}$$

Let us remark, that using the notation defined in [3] we can identify

$$-\cos\omega=S_L, \qquad \sin\omega=S_T, \qquad p_T\sin\omega\cos\varphi=\mathbf{p}_T\mathbf{S}_T, \tag{10}$$

which appear in definition of the TMDs [2]:

$$\frac{1}{2}\mathrm{tr}\left[\gamma^+\gamma_5\phi^q(x,\mathbf{p}_T)\right]=S_L g_1^q(x,p_T)+\frac{\mathbf{p}_T\mathbf{S}_T}{M}g_{1T}^{\perp q}(x,p_T). \tag{11}$$

Now, in analogy with Eq. (46) in [7] we define (note that $Pq/qS=-M/\cos\omega$):

$$w_1=M\nu\cdot w_S+\frac{M^2\nu}{\cos\omega}\cdot w_P, \qquad w_2=-\frac{M^2\nu}{\cos\omega}\cdot w_P, \tag{12}$$

which implies

$$g_k^q=\int \Delta G\left(p_0\right) w_k \delta\left(\frac{p_0+p_1}{M}-x\right)\frac{dp_1 d^2\mathbf{p}_T}{p_0}, \qquad k=1,2. \tag{13}$$

From Eqs.(8),(9) and the definition (12) we obtain

$$w_1=\frac{1}{2}\left(m+p_1\left(1+\frac{p_1}{m+p_0}\right)-p_T\tan\omega\left(1+\frac{p_1}{m+p_0}\right)\cos\varphi\right), \tag{14}$$

Apparently, the terms proportional to $\cos\varphi$ disappear in the integrals (13) and the remaining terms give structure functions $g_1, g_2$ defined by Eqs. (15),(16) in [6].

Now, the integration over $p_1$ and further procedure can be done in a similar way as for unpolarized distribution. First, to simplify calculation, we assume $m\to 0$. For $w_1$ we get

$$g_1^q(x)=\frac{1}{2}\int \Delta G_q(p_0)\left(1+\frac{p_1}{p_0}\right)\left(p_1-p_T\tan\omega\cos\varphi\right)\delta\left(\frac{p_0+p_1}{M}-x\right)\frac{dp_1 d^2\mathbf{p}_T}{p_0}. \tag{15}$$

The $\delta-$function is modified as

$$\delta\left(\frac{p_0+p_1}{M}-x\right)dp_1=\frac{\delta\left(p_1-\tilde{p}_1\right)dp_1}{x/\tilde{p}_0}, \tag{16}$$

where

$$\tilde{p}_1=\frac{Mx}{2}\left(1-\left(\frac{p_T}{Mx}\right)^2\right), \qquad \tilde{p}_0=\frac{Mx}{2}\left(1+\left(\frac{p_T}{Mx}\right)^2\right). \tag{17}$$

Modified $\delta-$function allows to simplify the integral

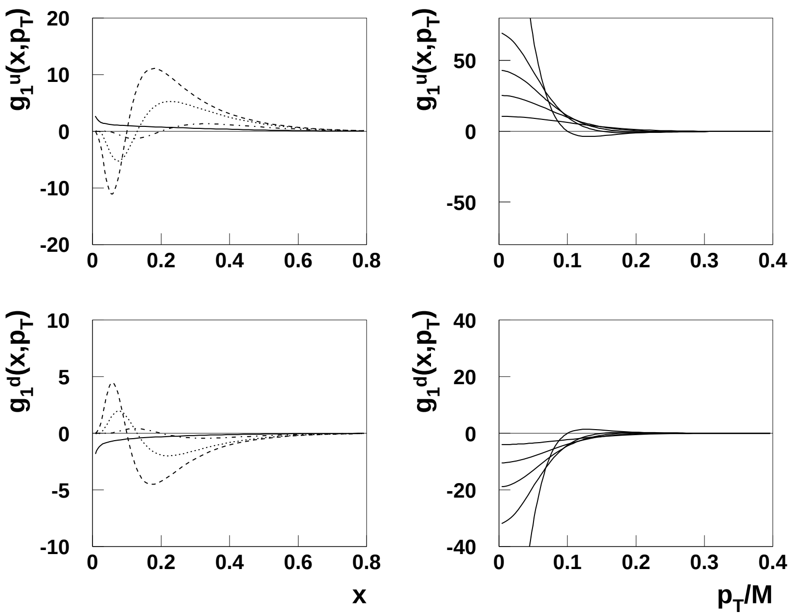


FIG. 3: Transverse momentum dependent polarized distribution functions for $u$ (*upper figures*) and $d-$quarks (*lower figures*). **Left part**: dependence on $x$ for $p_T/M = 0.10, 0.13, 0.20$ is indicated by dash, dotted and dash-dot curves; solid curve correspods to the integrated distribution $g_1^q(x)$. **Right part**: dependence on $p_T/M$ for $x = 0.10, 0.15, 0.18, 0.22, 0.30$ from top to down for $u-$quarks, and the same symmetrically for $d-$quarks.

$$g_1^q(x) = \frac{1}{2}\int \Delta G_q(\tilde{p}_0)\left(M\left(2x-\xi\right) - 2p_T \tan\omega\cos\varphi\right)\frac{d^2\mathbf{p}_T}{\xi}, \tag{18}$$

where

$$\xi = x\left(1 + \left(\frac{p_T}{Mx}\right)^2\right). \tag{19}$$

Now we define

$$\Delta q(x, \mathbf{p}_T) = \frac{1}{2}\Delta G_q\left(\frac{M\xi}{2}\right)\left(M\left(2x-\xi\right) - 2p_T\tan\omega\cos\varphi\right)\frac{1}{\xi}. \tag{20}$$

According to Eq. (40) in [6] we have

$$\Delta G_q\left(\frac{M\xi}{2}\right) = \frac{2}{\pi M^3\xi^2}\left(3g_1^q(\xi) + 2\int_\xi^1 \frac{g_1^q(y)}{y}dy - \xi\frac{d}{d\xi}g_1^q(\xi)\right). \tag{21}$$

After inserting to Eq. (20) one gets:

$$\begin{aligned}\Delta q(x, \mathbf{p}_T) \ &= \ \frac{1}{\pi M^2\xi^3}\left(3g_1^q(\xi) + 2\int_\xi^1 \frac{g_1^q(y)}{y}dy - \xi\frac{d}{d\xi}g_1^q(\xi)\right) \\ &\times\left(2x - \xi - 2\frac{p_T}{M}\tan\omega\cos\varphi\right).\end{aligned} \tag{22}$$

This relation allows us to calculate the distribution $\Delta q(x, \mathbf{p}_T)$ from a known input on $g_1^q(x)$. Further, it can be shown, that using the notation defined in Eqs. (10),(11), our result reads

$$-\cos\omega\cdot\Delta q(x, \mathbf{p}_T) = S_L g_1^q(x, p_T) + \frac{\mathbf{p}_T\mathbf{S}_T}{M}g_{1T}^{\perp q}(x, p_T), \tag{23}$$

where

$$g_1^q(x,p_T) = \frac{2x-\xi}{\pi M^2 \xi^3}\left(3g_1^q(\xi) + 2\int_\xi^1 \frac{g_1^q(y)}{y}dy - \xi\frac{d}{d\xi}g_1^q(\xi)\right), \tag{24}$$

$$g_{1T}^{\perp q}(x,p_T) = \frac{2}{\pi M^2 \xi^3}\left(3g_1^q(\xi) + 2\int_\xi^1 \frac{g_1^q(y)}{y}dy - \xi\frac{d}{d\xi}g_1^q(\xi)\right). \tag{25}$$

Apparently, both functions are related in our approach:

$$\frac{g_1^q(x,p_T)}{g_{1T}^{\perp q}(x,p_T)} = \frac{x}{2}\left(1-\left(\frac{p_T}{Mx}\right)^2\right) = \tilde{p}_1/M. \tag{26}$$

Finally, with the use of standard input [11] on $g_1^q(x) = \Delta q(x)/2$ we can obtain the curves $g_1^q(x,p_T)$ displayed in Fig. 3. Let us remark, that the curves change the sign at the point $p_T = Mx$. This change is due to the term

$$2x-\xi = x\left(1-\left(\frac{p_T}{Mx}\right)^2\right) = 2\tilde{p}_1/M \tag{27}$$

in relation (24). This term is proportional to the quark longitudinal momentum $\tilde{p}_1$ in the proton rest frame, which is defined by given $x$ and $p_T$. It means, that sign of the $g_1^q(x,p_T)$ is controlled by sign of $\tilde{p}_1$. On the other hand, the function $g_{1T}^{\perp q}(x,p_T)$ does not involve term, which changes the sign. The shape of both functions should be checked by experiment.

To conclude, we presented our recent results on relations between TMDs and PDFs. The study is in progress, further results will be published later.

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